

# Chosen indexes of technological assessment of mineral resources enrichment processes as a function of the concentrate's quality

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## ABSTRACT

In the article there are presented two applications of optimization problems of copper recovery from polish ores: the technological optimization based on a minimization of metal losses in tails and the economic one based on maximization of profit of the whole company consisted of mines, processing plants and copperworks. To achieve that, there were applied indexes based on average contents of copper in the feed ( $\alpha$ ), in the concentrate ( $\beta$ ) and in tails ( $\vartheta$ ). On the ground of the fact, that mass balance equation in geometry is represented by the hyperbolic parabola it was proved, that the dependence of the yield  $\gamma$  from the  $\beta$  can be well described by hyperbola. It allows to solve effectively the problems of optimization on the basis of nonlinear mathematical programming.

## 1. INTRODUCTION

The results of ores enrichment processes (the operating of the whole technological system in the plant) depend on the course of individual processes as well as on the quality of processed ore. It can be assumed, that every ore has its own washability profile what marks, that at specific preparation of it to enrichment process (obtaining of specific grain composition, liberation of mineral grains, application of suitable flotation reagents) it is generated a level of minerals recovery (recovery of metal  $\varepsilon$  and its content in concentrate the  $\beta$ ) as well as resulting from this

losses of metal. The change in the way of ore preparation or change in technological system both for operations of ore preparation and for enrichment operation, causes both the change in obtained enrichment effects and also in enrichment costs per mass of metal contained in the concentrate. It is possible to accept, that for specific ore the over natural increase in recovery of metal causes considerable higher increase in processing costs. The analogous situation treats to growth of the content of useful component  $\beta$  in the concentrate. The analysis of dependence of costs ( $C$ ) from  $\beta$ , that is the dependence  $C = C(\beta)$ , can be therefore here an essential matter.

According to the mass balance law, if the content  $\beta$  grows, then the content of useful component in tails  $\vartheta$  grows up also, which causes the economic and technological losses (not processing of possible to recovery amounts of metal and depositing them into sedimentary reservoir; there are then the elements of so-called robbery economy).

In conditions of copper processing plants in Poland it is essential the quantity of metal, which is transported from enrichment plants to copperworks, that is the relative quantity the  $\gamma\beta$  where the  $\gamma$  is the yield of the concentrate. For the application of mathematical programming methods it is useful to determine the function  $\gamma = \gamma(\beta)$ . In economic optimization of enrichment processes depending on qualification of minimum of target function, the function  $\gamma = \gamma(\beta)$  is used, which leads to reduction of number of variables

and allows for precise introduction of numerical limitations for the  $\beta$  value. Costs become however non-linear complex function of the  $\beta$  variable, that is  $C = C(\gamma(\beta))$ .

Different solutions connecting mentioned variables together and their application in searching of optimal technological and economic solutions will be therefore presented in the article.

## 2. TECHNOLOGICAL OPTIMIZATION OF POLISH COPPER ORES ENRICHMENT PROCESSES

Under the definition of technological optimization of enrichment processes systems we understand such determination of content of useful component in concentrate, which leads to the minimization of its losses in tails, taking into account the specific technology and type of enriched ore. On the problem of so defined efficiency paid attention Taggart (Taggart 1948), and then developed it Jowett (Jowett 1975) introducing following coefficient

$E = (\text{recovered quantity of component} / \text{recoverable quantity of component}) 100\%$

and also more detailed descriptions of above coefficient. Continuing these considerations Madej (Madej 1978) introduced the function of unit losses in following form

$$g(\alpha, \beta, \vartheta) = \frac{1 - \varepsilon}{\beta - \alpha} = \frac{\eta}{\beta - \alpha} = \frac{\vartheta}{(\beta - \vartheta)\alpha} \quad (1)$$

In laboratory and semi-industrial scale there are conducted investigation over final products of enrichment process that is over the concentrate and tails. There are executing flotation analyses with using the Dell's technique (Dell 1953), which depends on dividing of useful component from gangue, according to the level of the grain surface hydrofobization. Through gradual adding of flotation reagents and stage collecting of foam product, there were received several fractions. Next received fractions have smaller and smaller content of useful component, because the richest

grains come to the foam product most quickly, and the grains with respectively smaller content of useful component come to the foam product in turn, during the course of experiment. First fractions in described experiment will contain the most of the useful component, as it grows a serial number of separated fraction, the content of useful component in it diminishes itself. Referring to the grain size analysis in individual fractions, it is possible to examine the content of useful component and to determine its yield.

It turns out that represented by points  $(\beta - \alpha; g(\alpha, \beta, \vartheta))$  curve can be well described with parabola function, that is

$$g(\alpha, \beta, \vartheta) = \frac{\vartheta}{(\beta - \vartheta)\alpha} = a(\beta - \alpha)^2 + b(\beta - \alpha) + c \quad (2)$$

where a parameter is greater than zero. The function  $g(\alpha, \beta, \vartheta)$  achieves then minimal value for definite value  $(\beta - \alpha)$ . This is the indication of reasonable quality of concentrate at minimum relative technological price (the minimum metal

losses). That value equals  $(\beta - \alpha)_{opt} = -\frac{b}{2a}$ .

In Non-Ferrous Metals Institute in Gliwice there were led specific investigations in this range, and for three copper ore processing plants existing in Poland, following results were obtained:

for  $\alpha \cong 1,3\%$  :  $\beta_{opt} = 15,1 \div 16,8\%$   
 for  $\alpha \cong 1,9 \div 2,0\%$  :  $\beta_{opt} = 26 \div 27\%$   
 for  $\alpha \cong 2,03 \div 2,77\%$  :  $\beta_{opt} = 25 \div 29\%$

It is proper to pay the attention, that the difference  $(\beta - \alpha)$  characterizes the effect of enrichment better than single value  $\beta$ , losses are also calculated per unit of  $(\beta - \alpha)$  value. The technological optimization can be therefore treated as the best possible metal recovery (the useful component) from ore.

## 3. ECONOMIC OPTIMIZATION OF ENRICHMENT PROCESSES AND METALLURGIC PROCESSING OF COPPER

By economic optimization of electrolytical copper productions processes we will understand the

maximization of target function in following form

$$P = I - C$$

where:

P: the long-term profit of the plant

I: long-term income from selling of the metal

C: cost of metal production

Incomes and costs depend on quantity of concentrate tones with content  $\beta$ , enriched in processing plants and then transported and altered in copperworks. For methods of mathematical programming leading to maximization of target function in presence of limitations, the determination of function  $\gamma$  in following form:  $\gamma = \gamma(\beta)$ , where  $\gamma$  is the yield of concentrate, is an essential problem. It is possible to achieve it with using approximation resulting from mass balance equation (the modeling method), or by determination of this dependence on the ground of real data describing the work of processing plants (experimental method).

If we treat the mass balance equation as a dependence of three independent variables  $\beta$ ,  $\gamma$  and  $\vartheta$  (at fixed  $\alpha$  value), and record it in following form (Tumidajski Saramak 2002)

$$100\alpha = \gamma\beta + (1 - \gamma)\vartheta \quad (3)$$

then its geometrical form is the segment of hyperbolic parabola introduced in fig 1.

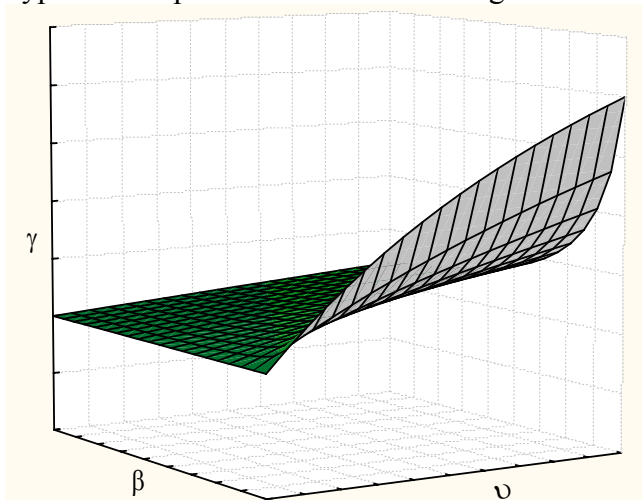


Figure 1. Functional dependence of yield  $\gamma$  from  $\beta$  and  $\alpha$ .

Real course of enrichment process is illustrated by some curve lying on the surface, which takes into account the mineralogical composition of ore as well as washability of the feed. In order to receive the dependence  $\gamma(\beta)$  as one variable function ( $\beta$ ) we may project this surface on ( $\beta$ ;  $\gamma$ ) plane (fig.2.).

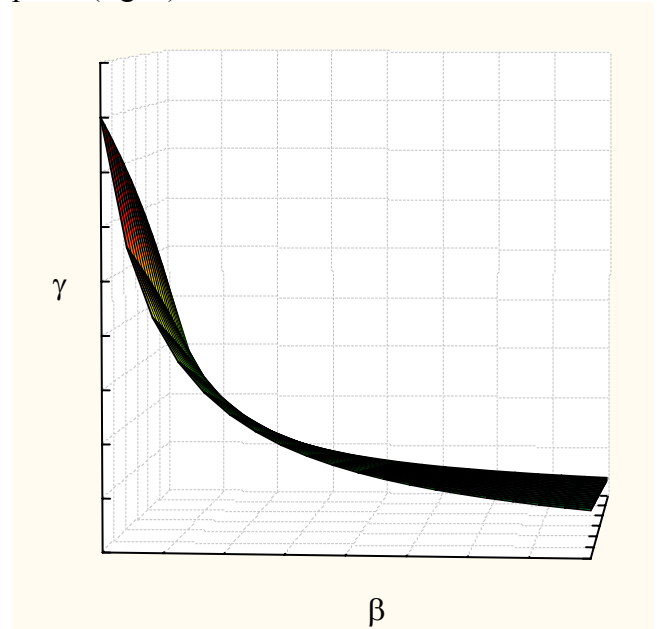


Figure 2. Functional dependence of yield  $\gamma(\beta)$  (a project of fig.1. on ( $\beta$ ;  $\gamma$ ) plane).

It is easy to notice that the dependence is easily to describe with hyperbole or parabola, while for  $\beta$  greater than 15% and smaller than 35% dependence has mainly linear approximation (Saramak 2004).

Let's assume therefore, that the curve of yield  $\gamma(\beta)$  can be described by formula

$$\gamma = \frac{a}{\beta} + b \quad (4)$$

where:

a,b: parameters.

From the viewpoint of technological process on the hyperbole (4) lie two extreme points, namely point A and point B. Point A, in which yield equals 100%, describes the extreme case when the enrichment process does not exist and concentrate makes up the whole ore (the content

of useful component in concentrate equals  $\alpha$ ). The point B on hyperbole is also the extreme case for enrichment process when all compounds of useful mineral pass on to concentrate (the theoretical ideal enrichment). The value of yield in this case equals the  $\gamma_t$  with content of metal  $\beta_t$ .

It easy to notice that both points of hyperbole described above will be different in dependence on profiles of enriched ores. In order to apply model (4) it is then essential to obtain following information about the ore:

- average the content of useful component,
- lithologic composition,
- average composition of useful mineral compounds.

There were determined points A and B of  $\gamma(\beta)$  function for individual processing plants in some accepted time period (Table 1)

Tab.1 Hypothetical co-ordinates of A and B points of hyperboles (2) and figures of function  $\gamma(\beta)$  for individual processing plants.

	Lubin	Polkowice	Rudna
Co-ordinates of A point	(1,18; 100)	(1,98; 100)	(2,31; 100)
Co-ordinates of B point	(52,14; 2,26)	(67,73; 2,92)	(63,45; 3,64)

Forms of  $\gamma(\beta)$  functions for specific processing plants will be then following:

$$\gamma = \frac{118,004}{\beta} - 0,0032 \quad \text{- for Lubin}$$

$$\gamma = \frac{198,007}{\beta} - 0,0035 \quad \text{- for Polkowice}$$

$$\gamma = \frac{231,002}{\beta} - 0,0007 \quad \text{- for Rudna}$$

The presented model is non-linear one. The parameter  $\alpha$  in it changes in random way, according to the nature of ore. In coursing of more detailed investigations that random changeability of parameter  $\alpha$  should be considered through regarding of both average and border values in the model.

On the ground of data collected in polish copper ore processing plants, with using of

regression method, there was determined a following dependence (Skorupska, Saramak 2005)

$$\beta = c\gamma^d \quad (5)$$

where:

c, d: parameters.

Values of coefficients in equation (5) are presented in Table 2.

Tab. 2 Values of coefficients in equation (3) for individual processing plants

plant \ parameter	Lubin	Polkowice	Rudna
d	75,70	123,87	148,26
c	-0,83	-0,84	-0,89

The convergence of approximation results with real ones presents fig. 3. Additionally fig.4. presents results of modeling, described in Table 2. Convergence of both methods of determining the dependence  $\gamma = \gamma(\beta)$  it is very high, the dependencies, obtained with using of two independent methods, practically cover themselves.

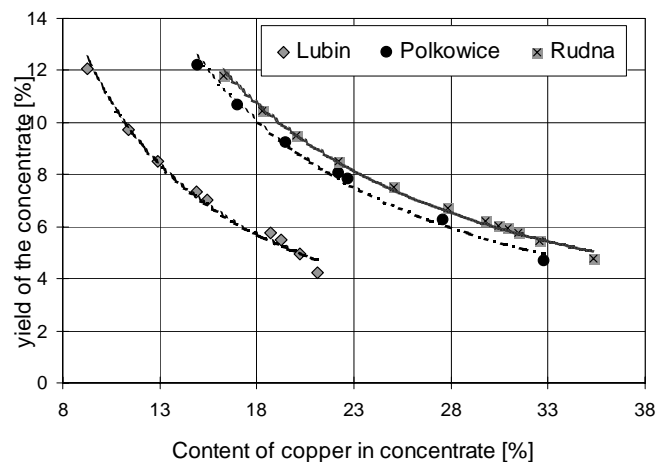


Figure 3. Dependencies  $\gamma = \gamma(\beta)$  obtained experimentally

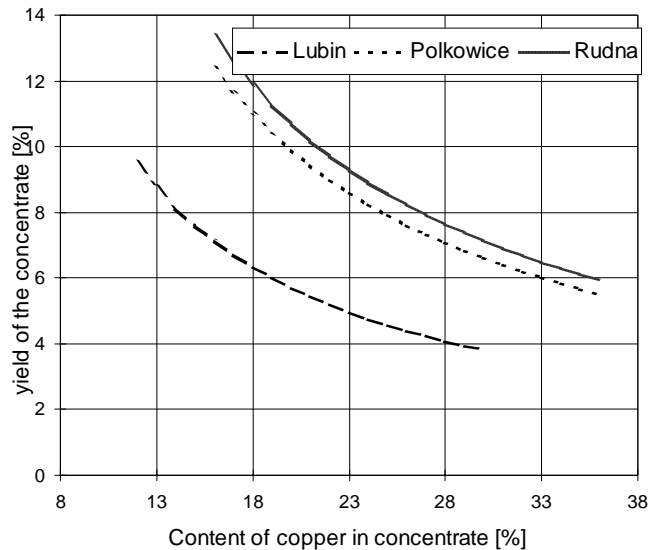


Figure 4. Dependencies  $\gamma = \gamma(\beta)$  obtained with using of modeling

#### 4. CONCLUSIONS

Introduced formulas enable to determine the enrichment strategy for Polish copper ores in dependence on both prices of copper at the world exchanges and variable profiles of ores (in different operating periods). Thanks to use of dependence  $\gamma = \gamma(\beta)$  as supplement of nonlinear mathematical programming problems, the limitations dependent only on  $\beta$  value occurred. Additionally, it occurred also limitations determining the quality of feed towards metallurgical processes namely: flash smelting processes and suspensional ones.

Approach presented in the article facilitates first of all the verification of optimizational problems with using of specific software (GAMS, AIMMS). The new method of verification of the issue is using of the genetic algorithms (Tumidajski and others 2005, Michalewicz 2003).

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